

IMPROVEMENT OF TILE DRAINS BY USING MOLE DRAINS FOR  
SOIL SUBJECTED TO ARTESIAN WATER TABLE.

By

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تحسين المصارف المغطاة باستخدام المصارف المثقلة في الاراضي المعرضة لضغوط بيزومترية.

خلاصة - في هذا البحث يقدم الباحث نظاما جديدا لصرف الاراضي التي تعاني من تسرب المياه من أسفل الى أعلى وذلك باستخدام نظام المصارف المثقلة كمصارف مساعدة للمصارف المغطاة والموجودة فعلا وذلك للتحكم في مستوى المياه الارضية فوق منسوب المصارف المغطاة. وقد تم الوصول الى استنتاج معادلات رياضية عامة جديدة لايجاد دالة الجهد المركب (  $W$  ) ودالة جهد السرعة (  $B$  ) ودالة السريران (  $\psi$  ). كما تم استنباط معادلتين جديدتين لحساب كميات التصريف الناتجة من كل مصرف من المصارف المثقلة المساعدة (  $Q_{m}$  ) والمصارف المغطاة (  $Q_p$  ). وقد شمل هذا البحث أيضا دراسة تأثير المتغيرات المختلفة على كميات التصريف الناتجة من كل مصرف لكل من المصارف المثقلة المساعدة والمصارف المغطاة. وهذه المتغيرات هي: المسافة الرأسية بين خط المصارف المثقلة المساعدة وخط المصارف المغطاة (  $b$  ) ، وارتفاع مستوى المياه الارضية المطلوبة بين مصرفين متتاليين من المصارف المغطاة (  $H$  ) ، وسك الطبقة الطينية أسفل خط المصارف المغطاة (  $D$  ) ، والضاغط البيزومتري (  $h_0$  ) ، والمسافة الأفقية بين مصرفين متتاليين من المصارف المغطاة (  $L$  ). وقد بين الباحث تلك التأثيرات برسم مجموعة من المنحنيات التوضيحية.

ABSTRACT - The author presents in this paper a new development for the problem of an agricultural soil subjected to an upward potential gradient for an existing tile drainage system by using mole drains. The case taken into account herein is that of a heavy clay layer underlain by a highly permeable aquifer of high piezometric head. The problem is hydrodynamically treated using the theory of complex functions and the theory of images. The complex potential, the velocity potential and stream functions are established. Formulas by means of which velocity components at any point may be calculated are provided. New discharge formulas for tile drains and mole drains are established. A complete study of the effect of the various parameters on the discharge per mole drain and per tile drain is presented.

#### INTRODUCTION

When an agricultural soil of a low hydraulic conductivity overlies a highly permeable aquifer of high piezometric head, artesian pressure causes an upward seepage flow in the agricultural soil. The control of water table levels by mole or tile drains has been investigated by many researchers such as: Vedecnikov [1], Kirkham [2], Hooghoudt [3], Hammad [4, 5 & 7], and Hathoot [8, 9, 10, 11, 12 & 13]. In their studies, they have determined the relation between the water table height and the spacing between drains for a given piezometric head inducing steady rate of upward flux. On the assumption that the ground surface is represented by an impervious layer and applying the method of images, Muskat [14] attempted the above problem. Hammad's solution [7] of the problem has been according to a hydrodynamical

treatment based on the complex functions and conformal mapping. All investigators attacked the problem considering one system of tile or mole drainags.

In the present paper, the problem of a tile drainage system which is assisted by a system of mole drains is treated for a clayey soil underlain by a highly permeable aquifer of high piezometric head. Figure(1) represents the geometry of the problem.

**BASIC ASSUMPTIONS AND BOUNDARY CONDITIONS.**

The basic assumptions employed in this paper are as follow:

- 1- Tile drains are running full but mole drains are running nearly empty.
- 2- Tile drains are equal in strength (M) and also, mole drains are equal in strength (m).
- 3- The soil is regarded homogeneous and isotropic. It is also assumed that there is no drainable water above the water table, which implies that a soil element fills or drains instantaneously to its final water content as the water table rises or falls through the element.
- 4- On the free water surface, the pressure is atmospheric.
- 5- the normal gradient of  $\psi$  disappears along the vertical lines of symmetry, (shown in Fig. 1).
- 6- The dotted vertical lines of Fig.(1) are lines of symmetry and therefore, there can be no flow across them.

**HYDRODYNAMICAL ANALYSIS:**

The system of tile drains shown in Fig.(1), may be represented by an infinite number of equidistant point sinks located at tile drain centres. The system of mole drains may be also represented by an infinite number of equidistant point sinks located at mole drain centres, Fig.(2). To represent the effect of the clay-sand and gravel interface, which is an equipotential line, a similar fictitious system of sources is assumed as shown in Fig.(2).

The complex potential of point sinks is

$$W1 = M \ln \sin \frac{\pi z}{L} + C1 \quad \dots\dots(1)$$

where M is the point sink strength and C1 is a real constant.

The complex potential of point sources is

$$W2 = -M \ln \sin \frac{\pi}{L} (z + 2iD) + C2 \quad \dots\dots(2)$$

The complex potential of five mole drains for one tile drain with the same spacing is given as follows:

$$W3 = m \ln \sin \frac{\pi}{L} (z-ib) + m \ln \sin \frac{\pi}{L} (z-a-ib) + m \ln \sin \frac{\pi}{L} (z+a-ib) + m \ln \sin \frac{\pi}{L} (z-2a-ib) + m \ln \sin \frac{\pi}{L} (z+2a-ib) + C3 \quad \dots\dots(3)$$

where m is the point sink strength.

The complex potential of five point sources is:

$$W4 = -m \ln \sin \frac{\pi}{L} (z+2iD+ib) - m \ln \sin \frac{\pi}{L} (z-a+2iD+ib) - m \ln \sin \frac{\pi}{L} (z+a+2iD+ib) - m \ln \sin \frac{\pi}{L} (z-2a+2iD+ib) - m \ln \sin \frac{\pi}{L} (z+2a+2iD+ib) + C4 \quad \dots\dots(4)$$

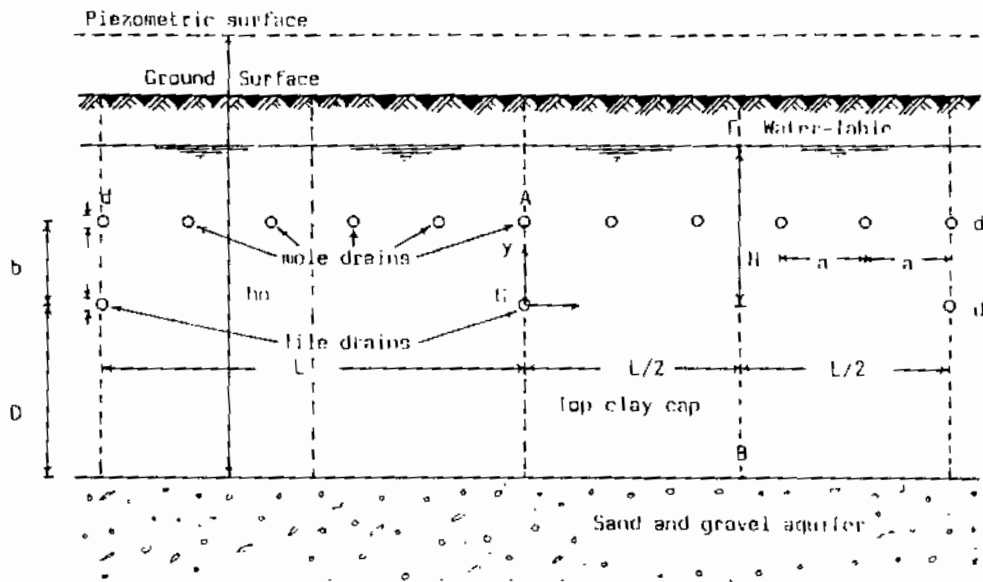


fig.(1): Hydrological section.

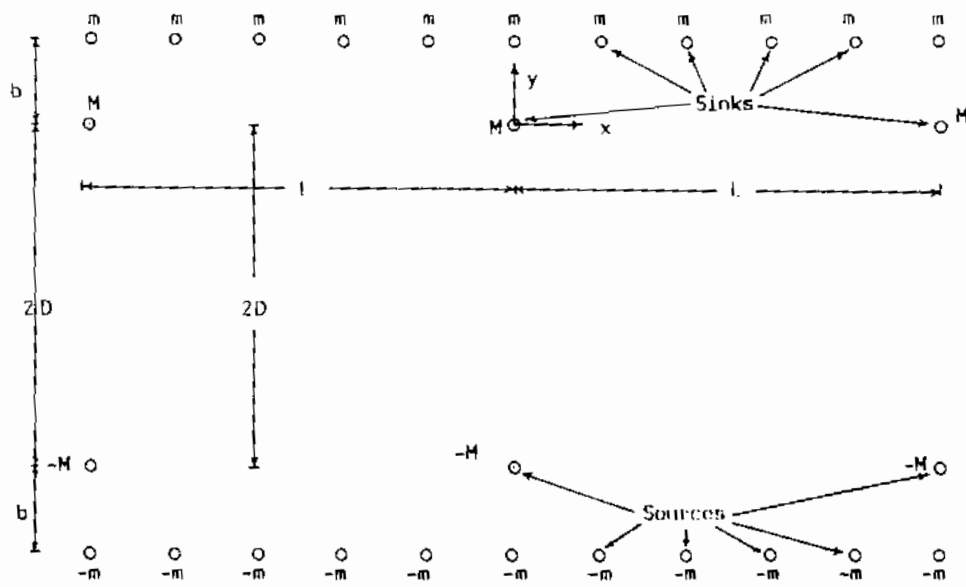


Fig. (2): Mathematical model.

where C2, C3 and C4 are real constants.

Therefore, the complex potential of the four systems is

$$W = W1 + W2 + W3 + W4 + C \quad \dots\dots(5)$$

where C is again a real constant.

Substituting  $Z = X + iy$ , where  $i = \sqrt{-1}$ , and simplifying:

$$\begin{aligned}
 W = & +M \ln(\sin \pi X/L \cdot \cosh \pi y/L + i \cos \pi X/L \cdot \sinh \pi y/L) \\
 & -M \ln(\sin \pi X/L \cdot \cosh \pi(y+2D)/L + i \cos \pi X/L \cdot \sinh \pi(y+2D)/L) \\
 & +m \ln(\sin \pi X/L \cdot \cosh \pi(y-b)/L + i \cos \pi X/L \cdot \sinh \pi(y-b)/L) \\
 & +m \ln(\sin \pi(x-a)/L \cdot \cosh \pi(y-b)/L + i \cos \pi(x-a)/L \cdot \sinh \pi(y-b)/L) \\
 & +m \ln(\sin \pi(x+a)/L \cdot \cosh \pi(y-b)/L + i \cos \pi(x+a)/L \cdot \sinh \pi(y-b)/L) \\
 & +m \ln(\sin \pi(x-2a)/L \cdot \cosh \pi(y-b)/L + i \cos \pi(x-2a)/L \cdot \sinh \pi(y-b)/L) \\
 & +m \ln(\sin \pi(x+2a)/L \cdot \cosh \pi(y-b)/L + i \cos \pi(x+2a)/L \cdot \sinh \pi(y-b)/L) \\
 & -m \ln(\sin \pi X/L \cdot \cosh \pi(y+b+2D)/L + i \cos \pi X/L \cdot \sinh \pi(y+b+2D)/L) \\
 & -m \ln(\sin \pi(x-a)/L \cdot \cosh \pi(y+b+2D)/L + i \cos \pi(x-a)/L \cdot \sinh \pi(y+b+2D)/L) \\
 & -m \ln(\sin \pi(x+a)/L \cdot \cosh \pi(y+b+2D)/L + i \cos \pi(x+a)/L \cdot \sinh \pi(y+b+2D)/L) \\
 & -m \ln(\sin \pi(x-2a)/L \cdot \cosh \pi(y+b+2D)/L + i \cos \pi(x-2a)/L \cdot \sinh \pi(y+b+2D)/L) \\
 & -m \ln(\sin \pi(x+2a)/L \cdot \cosh \pi(y+b+2D)/L + i \cos \pi(x+2a)/L \cdot \sinh \pi(y+b+2D)/L) \\
 & +C \quad \dots\dots(6)
 \end{aligned}$$

Substituting  $W = \phi + i\psi$ , where  $\phi$  is the velocity potential and  $\psi$  is the stream function, equating real to real and imaginary to imaginary on both sides of the above equation and simplifying we get:

$$\begin{aligned}
 \phi = & M/2 \cdot \ln((\sin^2 \pi X/L + \sinh^2 \pi y/L)/(\sin^2 \pi X/L + \sinh^2 \pi(y+2D)/L)) \\
 & +m/2 \cdot \sum_0^n \ln((\sin^2 \pi(x-na)/L + \sinh^2 \pi(y-b)/L)/(\sin^2 \pi(x-na)/L + \sinh^2 \pi(y+b+2D)/L)) \\
 & +m/2 \cdot \sum_1^n \ln((\sin^2 \pi(x+na)/L + \sinh^2 \pi y-b/L)/(\sin^2 \pi(x+na)/L + \sinh^2 \pi(y+b+2D)/L)) \\
 & +C \quad \dots\dots(7)
 \end{aligned}$$

$$\begin{aligned}
 \psi = & M(\tan^{-1}(\cot \pi X/L \cdot \tanh \pi Y/L) - \tan^{-1}(\cot \pi X/L \cdot \tanh \pi(y+2D)/L)) \\
 & +m \sum_0^n \tan^{-1}(\cot \pi(x-na)/L \cdot \tanh \pi(y-b)/L) - m \sum_1^n \tan^{-1}(\cot \pi(x+na)/L \cdot \tanh \pi(y+b+2D)/L) \\
 & \dots\dots(8)
 \end{aligned}$$

where  $n = ((L/a)-1)/2$

**VELOCITY CONSIDERATIONS**

From textbooks on hydrodynamics [ 6], velocity components at any point in the flow field are given by:

$$u = -\partial \phi / \partial x \quad \dots\dots(9) \quad \text{and} \quad v = -\partial \phi / \partial y \quad \dots\dots(10)$$

in which u and v are the velocity components in the X and y directions respectively.

Differentiating Eq.(7), partially, with respect to x and simplifying; the horizontal velocity component u is given by:

$$\begin{aligned}
 u = & -M \pi / 2L ((\sin 2 \pi X/L / (\sin^2 \pi X/L + \sinh^2 \pi y/L)) - (\sin 2 \pi X/L / \\
 & (\sin^2 \pi X/L + \sinh^2 \pi(y+2D)/L)))
 \end{aligned}$$

$$\begin{aligned}
 & -\pi/2L \sum_0^n ((\sin^2 \pi(x-na)/L)/(\sin^2 \pi(x-na)/L + \sinh^2 \pi(y-b)/L)) - (\sin^2 \pi(x-na)/L / \\
 & (\sin^2 \pi(x-na)/L + \sinh^2 \pi(y+b+2D)/L)) \\
 & -\pi/2L \sum_1^n ((\sin^2 \pi(x+na)/L)/(\sin^2 \pi(x+na)/L + \sinh^2 \pi(y-b)/L)) - (\sin^2 \pi(x+na)/L / \\
 & (\sin^2 \pi(x+na)/L + \sinh^2 \pi(y+b+2D)/L)) \dots\dots(11)
 \end{aligned}$$

similarly, differentiating Eq.(7) partially with respect to y and simplifying, the vertical velocity component V is given as:

$$\begin{aligned}
 V = & -M\pi/2L ((\sinh^2 \pi y/L)/(\sin^2 \pi x/L + \sinh^2 \pi y/L)) - (\sinh^2 \pi(y+2D)/L / \\
 & (\sin^2 \pi x/L + \sinh^2 \pi(y+2D)/L)) \\
 & -\pi/2L \sum_0^n ((\sinh^2 \pi(y-b)/L)/(\sin^2 \pi(x-na)/L + \sinh^2 \pi(y-b)/L)) - (\sinh^2 \pi(y+b+2D)/L / \\
 & (\sin^2 \pi(x-na)/L + \sinh^2 \pi(y+b+2D)/L)) \\
 & -\pi/2L \sum_1^n ((\sinh^2 \pi(y-b)/L)/(\sin^2 \pi(x+na)/L + \sinh^2 \pi(y-b)/L)) - (\sinh^2 \pi(y+b+2D)/L / \\
 & (\sin^2 \pi(x+na)/L + \sinh^2 \pi(y+b+2D)/L)) \dots\dots(12)
 \end{aligned}$$

from Eqs.(11) and (12) the flow velocity at a general point (x, y) can be obtained.

Inspection of Eq.(11) shows that for x = 0 and x = L/2 the horizontal component of velocity is zero. This satisfies an important boundary condition since x = ± L/2 are vertical lines of symmetry. Also, the horizontal component u is zero at y = -D. This satisfies the condition that the horizontal line y = -D is an equipotential.

DISCHARGE FORMULAS

The velocity potential of flow, φ, may be put in this form

$$\phi = K \left( \frac{P}{\rho g} + y \right) \dots\dots(13)$$

where K is the hydraulic conductivity of clay, P is the gauge pressure, ρ is the density of drained water and g is the acceleration due to gravity. From Eqs.(7) and (13), we have.

$$\begin{aligned}
 K \left( \frac{P}{\rho g} + y \right) = & H/2 \cdot \ln((\sin^2 \pi x/L + \sinh^2 \pi y/L)/(\sin^2 \pi x/L + \sinh^2 \pi(y+2D)/L)) \\
 & + H/2 \cdot \sum_0^n \ln((\sin^2 \pi(x-na)/L + \sinh^2 \pi(y-b)/L)/(\sin^2 \pi(x-na)/L \\
 & + \sinh^2 \pi(y+b+2D)/L)) + H/2 \cdot \sum_1^n \ln((\sin^2 \pi(x+na)/L + \sinh^2 \pi(y-b)/L)/ \\
 & (\sin^2 \pi(x+na)/L + \sinh^2 \pi(y+b+2D)/L)) + C \dots\dots(14)
 \end{aligned}$$

Applying Eq.(14) at point B (L/2, -D), we get

$$\begin{aligned}
 \therefore K (h_0 - D) & = 0 + 0 + 0 + C \\
 \therefore C & = K(h_0 - D) \dots\dots(15)
 \end{aligned}$$

Applying Eq.(14) at point G(0,0, d/2), where the pressure is atmospheric, one obtains.

$$\begin{aligned} \therefore K(d/2+D-h_0) &= M/2 \cdot \ln(\sinh^2 \pi d/2L / \sinh^2 \pi(d/2 + 2D)/L) \\ &+ m/2 \cdot \ln(\sinh^2 \pi(d/2-b)/L / \sinh^2 \pi(d/2+b+2D)/L) \\ &+ m \cdot \sum_1^n \cdot \ln((\sin^2 \pi na/L + \sinh^2 \pi(d/2-b)/L) / (\sin^2 \pi na/L + \\ &\quad \sinh^2 \pi(d/2 + b+2D)/L)) \dots\dots(16) \end{aligned}$$

The mole drain is assumed to run nearly empty. Applying Eq.(14) to point A(0.0, (b-d/2)) at the mole drain bottom where the pressure is atmospheric, the following is obtained.

$$\begin{aligned} \therefore K(b+D-h_0-d/2) &= M/2 \cdot \ln(\sinh^2 \pi(b-d/2)/L / \sinh^2 \pi(b+2D-d/2)/L) \\ &+ m/2 \cdot \ln(\sinh^2 \pi(-d/2)/L / \sinh^2 \pi(2b+2D-d/2)/L) \\ &+ m \cdot \sum_1^n \cdot \ln((\sin^2 \pi na/L + \sinh^2 \pi(-d/2)/L) / (\sin^2 \pi na/L + \\ &\quad \sinh^2 \pi(2b+2D-d/2)/L)) \dots\dots(17) \end{aligned}$$

Applying Eq.(14) to point F(L/2, H) on the free water surface, Fig.(1), it follows,

$$\begin{aligned} \therefore K(H+D-h_0) &= M/2 \cdot \ln(\cosh^2 \pi H/L / \cosh^2 \pi(H+2D)/L) \\ &+ m/2 \cdot \sum_0^n \cdot \ln((\cos^2 \pi na/L + \sinh^2 \pi(H-b)/L) / (\cos^2 \pi na/L + \sinh^2 \pi(H+b+2D)/L)) \\ &+ m/2 \cdot \sum_1^n \cdot \ln((\cos^2 \pi na/L + \sinh^2 \pi(H-b)/L) / (\cos^2 \pi na/L + \sinh^2 \pi(H+b+2D)/L)) \dots\dots(18) \end{aligned}$$

The above equations may be rewritten as follows:

$$K(d/2 + D - h_0) = M \delta_1 + m (\delta_2 + \delta_3) \dots\dots(19)$$

$$K(b + D - h_0 - d/2) = M \delta_4 + m (\delta_5 + \delta_6) \dots\dots(20)$$

$$K(H + D - h_0) = M \delta_7 + m (\delta_8 + \delta_9) \dots\dots(21)$$

in which

$$\begin{aligned} \delta_1 &= \frac{1}{2} \cdot \ln(\sinh^2 \pi d/2L / \sinh^2 \pi(d/2 + 2D)/L) \\ \delta_2 &= \frac{1}{2} \cdot \ln(\sinh^2 \pi(d/2-b)/L / \sinh^2 \pi(d/2+b+2D)/L) \\ \delta_3 &= \sum_1^n \cdot \ln((\sin^2 \pi na/L + \sinh^2 \pi(d/2-b)/L) / (\sin^2 \pi na/L + \sinh^2 \pi(d/2+b+2D)/L)) \\ \delta_4 &= \frac{1}{2} \cdot \ln(\sinh^2 \pi(b-d/2)/L / \sinh^2 \pi(b+2D-d/2)/L) \\ \delta_5 &= \frac{1}{2} \cdot \ln(\sinh^2 \pi(-d/2)/L / \sinh^2 \pi(2b+2D-d/2)/L) \\ \delta_6 &= \sum_1^n \cdot \ln((\sin^2 \pi na/L + \sinh^2 \pi(-d/2)/L) / (\sin^2 \pi na/L + \sinh^2 \pi(2b+2D-d/2)/L)) \end{aligned}$$

$$\begin{aligned} \delta_7 &= \frac{1}{2} \ln(\cosh^2 \pi H/L + \sinh^2 \pi (H+2D)/L) \\ \delta_8 &= \frac{1}{2} \sum_{n=0}^{\infty} \ln((\cos^2 \pi na/L + \sinh^2 \pi (H-b)/L)/(\cos^2 \pi na/L + \sinh^2 \pi (2D+H+b)/L)) \\ \delta_9 &= \frac{1}{2} \sum_{n=1}^{\infty} \ln((\cos^2 \pi na/L + \sinh^2 \pi (H-b)/L)/(\cos^2 \pi na/L + \sinh^2 \pi (H+b+2D)/L)) \end{aligned}$$

From Eqs.(19) and (21) we get

$$K(d/2+H+2D-2ho) = M(\delta_1 + \delta_7) + m(\delta_2 + \delta_3 + \delta_8 + \delta_9) \quad \dots\dots(22)$$

Also, from Eqs. (20) and (21) we get

$$K(H+2D+b-2ho-d/2) = M(\delta_4 + \delta_7) + m(\delta_5 + \delta_6 + \delta_8 + \delta_9) \quad \dots\dots(23)$$

The above equations may be again rewritten as follows,

$$\therefore K\sigma_1 = M\eta_1 + m\eta_2 \quad \dots\dots(24)$$

$$K\sigma_2 = M\eta_3 + m\eta_4 \quad \dots\dots(25)$$

in which

$$\sigma_1 = (d/2 + H + 2D - 2ho)$$

$$\sigma_2 = (H + 2D + b - 2ho - d/2)$$

$$\eta_1 = \delta_1 + \delta_7$$

$$\eta_2 = \delta_2 + \delta_3 + \delta_8 + \delta_9$$

$$\eta_3 = \delta_4 + \delta_7$$

$$\eta_4 = \delta_5 + \delta_6 + \delta_8 + \delta_9$$

From Eqs.(24) and (25) and solving for M and m we get,

$$M = K\theta \quad \dots\dots(26)$$

$$m = K(\sigma_1 - \eta_1 \cdot \theta) / \eta_2 \quad \dots\dots(27)$$

$$\text{where } \theta = (\sigma_1 \cdot \eta_4 - \sigma_2 \cdot \eta_2) / (\eta_1 \cdot \eta_4 - \eta_2 \cdot \eta_3)$$

Therefore, from Eq.(26), the discharge reaching each unit length of tile drain is given by

$$Q_T = 2\pi K\theta \quad \dots\dots(28)$$

From Eq.(27), the discharge reaching each unit length of mole drain is also given by;

$$Q_m = 2 \pi K (\theta_1 - \eta_1 \theta) / \eta_2 \dots\dots(29)$$

**EFFECT OF THE VARIOUS PARAMETERS ON THE DRAIN DISCHARGE:**

It is found from Eqs.(28) and (29) that the discharge per unit length of tile drain or mole drain is affected by certain parameters such as b, H, ho,.....etc.

In the following, we shall discuss the effect of the principal parameters on the discharge per unit length of tile drain and the discharge per unit length of mole drain.

**A- EFFECT OF VERTICAL SPACING, b. -**

The effect of vertical spacing, b, on drain discharge is illustrated in Fig.(3). As the vertical spacing, b, increases the discharge per unit length of tile drain increases but the discharge per unit length of mole drain decreases. The curve for tile drain is steep while for mole drain it is flat.

**B- EFFECT OF WATER HEAD, H.**

Figure (4) illustrates the relation between the discharge and the water table height. As the water head, H, increases the discharge per unit length of tile and mole drain decreases. This is because increasing H means a smaller required depression of water table from the original position.

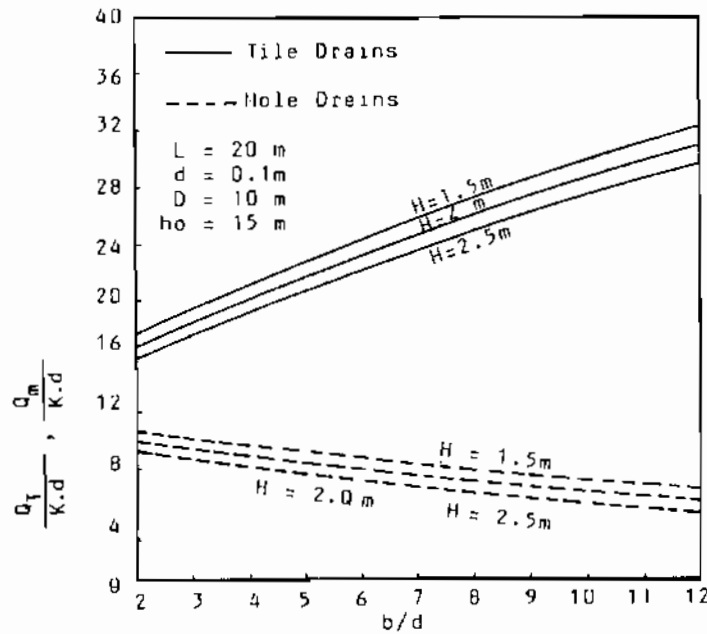


Fig.(3): Discharge per Drain Versus Spacing, b.



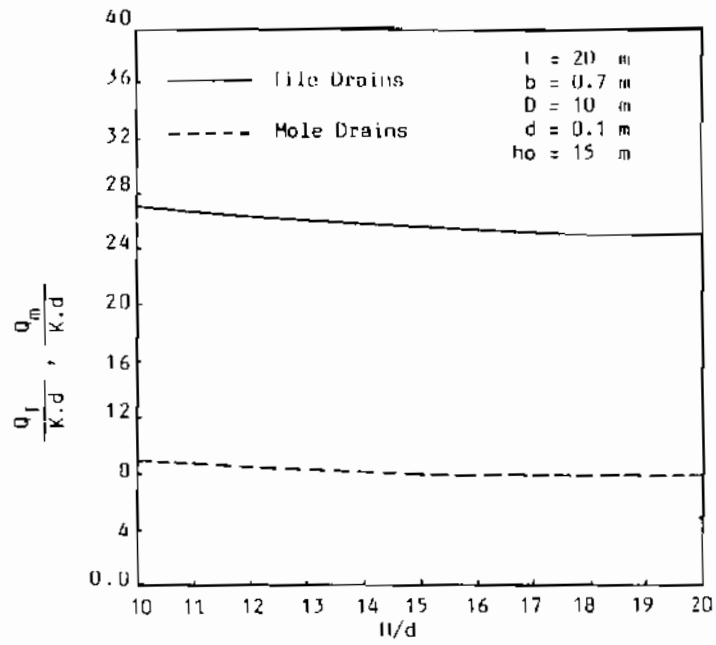


Fig.(4): Discharge per Drain Versus water head,  $h$ .

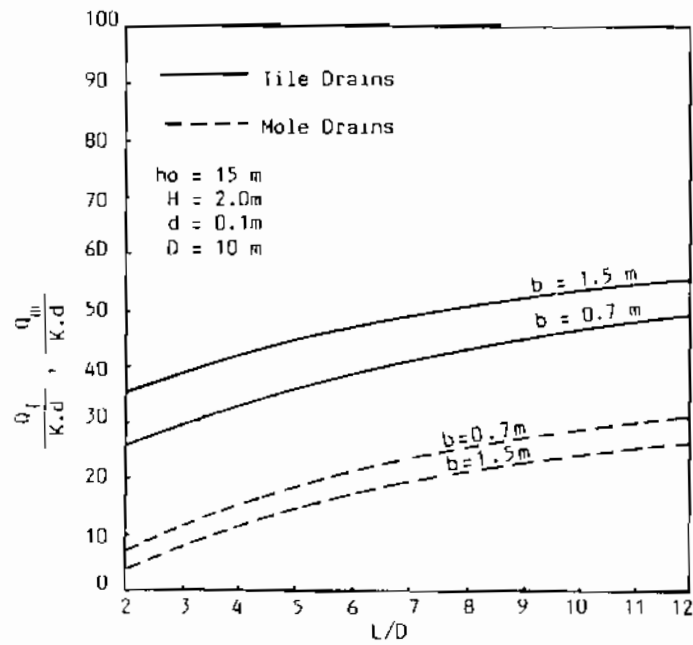


Fig.(5): Discharge per Drain Versus spacing,  $l$ .

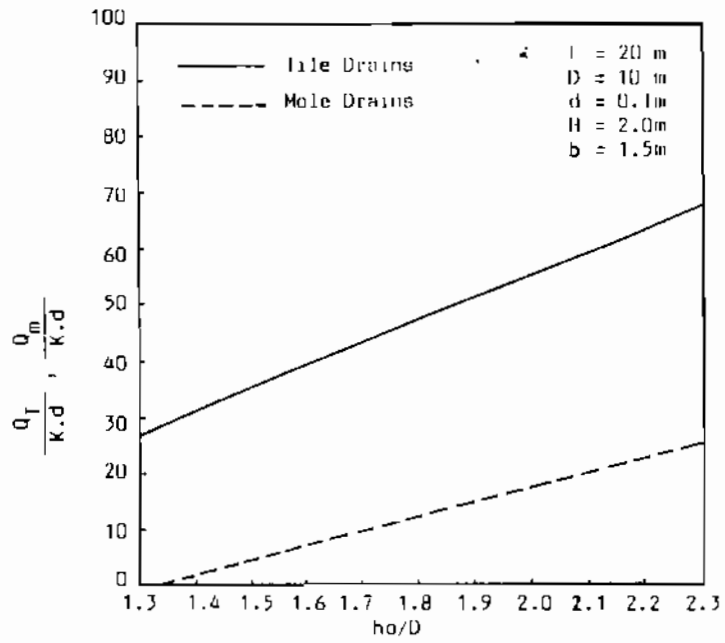


Fig.(6): Discharge per Drain Versus Piezometric head,  $h_o$ .

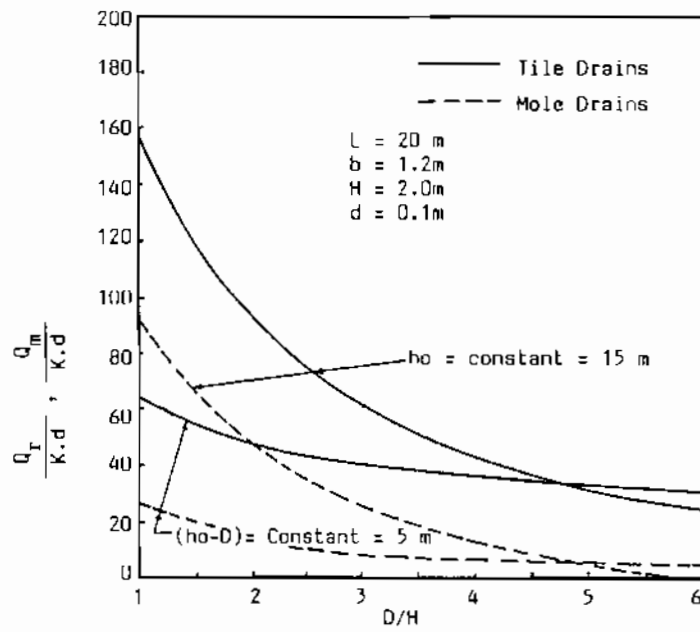


Fig.(7): Discharge per Drain versus soil thickness,  $D$ .

#### C- EFFECT OF SPACING, L.

Figure (5) shows that the discharge per unit length of tile drain and per unit length of mole drain increases when increasing the spacing L. Figure(5) is regarded as a discharge design chart in a non-dimensional form.

#### D- EFFECT OF THE PIEZOMETRIC HEAD, $h_0$ .

In Fig.(6) the relation between the discharge per unit length of tile drain and per unit length of mole drain and the piezometric head of the sand aquifer,  $h_0$ , is shown plotted. From this figure, it is evident that the discharge per unit length of both tile drain and mole drain increases on increasing the piezometric head,  $h_0$ .

#### E- EFFECT OF SOIL THICKNESS, D.

Figure (7) illustrates the effect of soil thickness below tile drains, D, on the discharge per unit length of both tile drain and mole drain. As the soil thickness increases the discharge per unit length of tile drain and per unit length of mole drain decreases. It is clearly seen that the curves are steeper at smaller values of soil thickness, D, and as soil thickness increases the curve gradually flattens.

#### CONCLUSIONS

A new improvement for an existing tile drainage system using mole drains to protect an agricultural clay layer, underlain by a sand and gravel aquifer of high piezometric pressure is established in this paper. The theory of complex functions and the theory of images are used to establish the velocity potential and the stream function. Velocity components at a general point in the flow field are also derived. Mathematical solution when checked by applying the velocity formulas is found to satisfy the imposed boundary conditions. The new discharge formulas for tile drains and mole drains are derived. The effect of several parameters, such as vertical spacing between mole drains and tile drains, height of water table, the piezometric head...etc. on the discharge per unit length of tile drain and per unit length of mole drain are studied. Graphs are provided to illustrate the relation between the discharge per unit length of drain and each of the involved parameters. From the above relations, it is evident that, the best economical achievement occurs when the small vertical spacing, b, is used.

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NOTATION

The following symbols are used in this paper:

- $u$  = spacing between two successive mole drains;
- $b$  = vertical spacing between lines of tile drains and mole drains;
- $d$  = drain diameter for both tile and mole drains;
- $D$  = depth of top clay cap below tile drains;
- $g$  = acceleration due to gravity;
- $h_0$  = piezometric head of sand and gravel aquifer;
- $H$  = height of water table above tile drains at the mid point between two successive tile drains;
- $i$  =  $\sqrt{-1}$  ;
- $K$  = Hydraulic conductivity of clay;
- $l$  = spacing between two successive tile drains;
- $m$  = strength of a point sink for mole drains;
- $M$  = strength of a point sink for tile drains;
- $n$  =  $((L/a)-1)/2$  ;
- $P$  = gauge pressure at any general point (x,y);
- $Q$  = discharge reaching each unit length of mole drains;
- $Q_T$  = discharge reaching each unit length of tile drains;
- $u$  = velocity component in the x-direction;
- $U$  = vertical downward steady streaming representing rain or irrigation water;
- $v$  = velocity component in the Y-direction,
- $W$  = complex potential =  $\Phi + i\psi$  ;
- $x, y$  = coordinates of any point in the field of motion;
- $z$  = complex number =  $x+iy$ ;
- $\Phi$  = the equipotential function;
- $\Psi$  = the stream function; and
- $\rho$  = water density.